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Accelerometer sensor for physical pendulum a complete formulation

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ABSTRACT*: We have led a compound pendulum analyze utilizing Arduino and a related two-pivot accelerometer sensor as an estimating gadget. We have indicated that the utilization of an accelerometer to gauge both outspread and orbital increasing speeds of the pendulum at various situations along its pivot offers the chance of playing out a more intricate examination contrasted with the standard investigation of the pendulum analyze. Along these lines, we have demonstrated that this traditional trial can prompt a fascinating and ease test in mechanics.*

I. INTRODUCTION

The physical pendulum experiment is the typical onetointroducethephysicsofoscillatingsystems. The usual aim of the analysis of this experiment istodeterminethependulumperiodanddamping factor by using an angular position sensor [1,2].

Wehaveconductedthependulumexperimentbyusingatwoaxisaccelerometersensor.Suchsensorhasalreadybeenusedb yFernandesetal.(2017)[3] but their study focused on the analysis of the time variation of the radial acceleration to investigate large-angle anharmonicoscillations.

Here,wehaveusedtheaccelerometertomeasure both radial and orbital accelerations of thependu-lumatdifferentpositionsalongitsaxis,whichoffers thepossibilityofperformingamorecomplexanal-ysis compared to the usual single measurement of the pendulum period.

Furthermore, we use a microcontroller and an associatedtwoaxisaccelerometersensortoacquire the data. Thus, we have used this simple and low costexperimentcomparedtothereadytousecom- mercial one to introduce a richer theory and data analysistoolsthatcanleadtoaninterestingexperiment inmechanics.

Wehavedescribedthetheoreticalanalysisofthis experimentandpresentanexampleofapossibleexperimental setup, the analysis of the measuredradialandorbitalaccelerationinordertoacquirethe momentofinertia,thecenterofmass,thedamping factorandtheperiodofthependulum.

II. EXAMPLE OF EXPERIMENTALSETUP

WeuseanArduino[4]andatwoaxisaccelerometersensorasmeasuringdevices.TheArduinoisaninterestingchoiceforan experiment,asitisaneasy- to-use and low-cost microcontroller, with a large user community. Even if Arduino was not initially developedasaphysicisttool,itcanbeusedinvari- ouscontextsofexperimentalphysicsactivities(e.g., see references[5–10]).

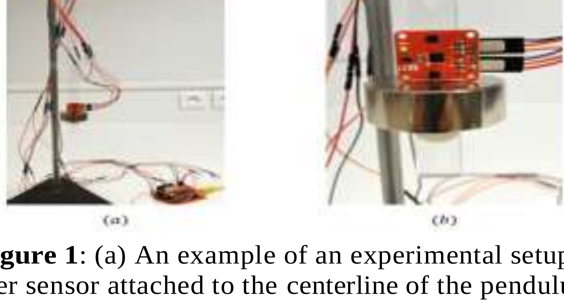

Figure 1: (a) An example of an experimental setup.
(b) An accelerometer sensor attached to the centerline of the pendulum, with one of its axes parallel to the centerline.

The experimental setup that we used here is shown in Fig. 1(a). The accelerometer sensor is a microelectromechanical inertial sensor which is preciselycalibratedbyusingthevalues+g,0gand
−g for each axis with $g = 9.8\ m\ s^{-2}$.
Thependulumusedinthisexperimentiscom-posed of a bar on which masses can beattachedto different positions. The accelerometerisattachedto the bar and positioned in such a way thatoneofits measurement axes lies parallel to thebar(Fig.1(b)). Special care should be taken sothatthewiresconnectingtheaccelerometertotheboardareflexibleenoughinordernottodampthependulum.Figure2showsanexampleofdataacquiredbythe accelerometer. The main features of thegraph

are:
- the radial acceleration measurement decreases and goes to g as t approaches the infinity.
- when the radial acceleration reaches itsmaxi- mum values, the orbital acceleration is essen- tially equal to zero (inset ofFig. 2).

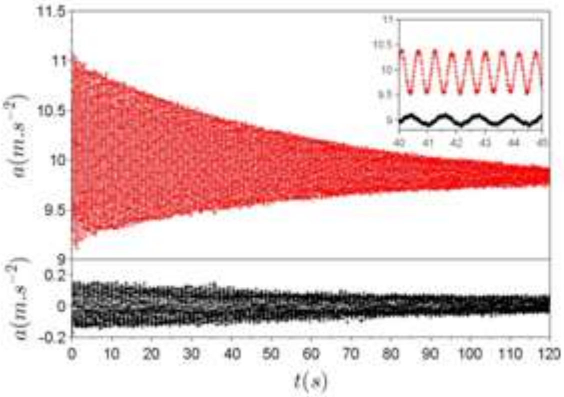

Figure 2: Radial (red) and orbital (black) accelerations measured by the accelerometer obtained with the ex- perimental setup displayed in Fig. 1. Inset shows the temporal evolution of both accelerations between 40 s and 45 s. In order to have both curves on the same plot, orbital acceleration is shifted by a constant offset of 9 in theinset. the period of the radial acceleration oscilla- tions is twice that of the orbital acceleration oscillations (inset of Fig. 2).

In the next section, we will present the theory that explains these main features.

III. THEORY

i. Expression of the acceleration compo- nents measured by the accelerometer sensor

Figure 3 shows a pendulum sketch with notations that will be used throughout this paper. O is the pivot point, tt is the pendulum mass center and A the accelerometer position. L and r stands for the distance between O and tt and O and A, respec- tively. We note $I_\lambda$the moment of inertia of the pendulum about the Oz axis.
Applyingtheangularmomentumtheoremtothe pendulum and considering viscous damping leads to:

- theradialaccelerationisasymmetricaboutthe

$$I_\lambda\ddot{\theta}=-MgL\sin\theta-\gamma\dot{\theta} \quad (1)$$

straight line a = g contrary to the orbital acceleration that is symmetric about the line a = 0.
where θ is the angle between the pendulumaxis Ott and the vertical axis, M is the mass of the sys- temandγisthecoefficientoffriction.Wenote small damping ($\kappa \ll \omega$) and the small angle ap- proximation ($\theta_0 \ll \frac{\pi}{2}$), which leads from Eq. (2)
to:

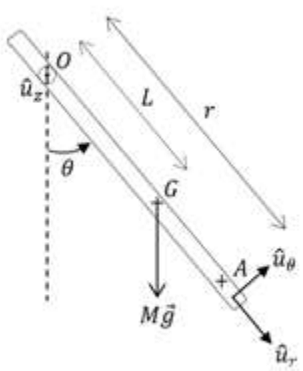

Figure 3: Sketch of the pendulum and notations used in the text. The pivot is at O, the center of mass at tt and the accelerometer sensor is at A.

$I_\lambda = \alpha ML2$ where the numerical factor α depends on the type of pendulum ($\alpha = 1$ for a simple pen-
dulum and $\alpha f= 1$ for a physical pendulum). We

2/

$\theta = \theta 0 e^{-\kappa t}\cos(\omega t)$ (8) 2

$\dot{\theta} = -\omega\theta 0 e^{-\kappa t}\sin(\omega t)$ (9)

$\ddot{\theta} = -\omega 2\theta 0 e^{-\kappa t}\cos(\omega t)$ (10)

with the pendulum angular frequency $\omega=$

2 2 g √ √

0 $\omega - \kappa$ and $\omega =$

ation components measured by the accelerometer

for $\kappa << \omega$ and $\theta 0 << ^{\pi}$ are given by: $/\alpha L$.Thus,theaccelero

2

$a_r = r\omega 2\theta 2 e^{-} 2^{\kappa t}\sin 2(\omega t)$

κt . 0 Σ (11)

0 $+ g\cos\theta e$

$a_\theta = -2\kappa r\omega\theta 0 e^{-\kappa t}\sin(\omega t)$ $\cos(\omega t)$

$+ \frac{g\cdot r}{} - \alpha \ \sin\dot{\ }\theta e^{-\kappa t}\cos(\omega t)$ Σ Σ (12)

introduce the damping factor $\kappa = ^{\gamma}$ α L 0

λ

g

to obtain:

With $\theta 0 << ^{\pi}$, we can further approximate these

$\ddot{\theta} = -\frac{}{\alpha L}\sin\theta - 2\kappa\dot{\theta}$ ——

(2)

two expressions by:

Forces acting on the proof mass (m_{sensor}) inside the accelerometer are its weight and the inertial forceduetoitsmovement.Thus,theradialandor-

$a_r = r\omega 2\theta 2 e^{-} 2^{\kappa t}\sin 2(\omega t)$ 0

$+ g\dot{\ }1 - \theta 2 e^{-} 2^{\kappa t}\cos 2(\omega t)$ Σ 0

(13)

bitalcomponentsoftheseforcesinthenon-inertial reference frame of the pendulum are givenby:

$F_r = m_{sensor} r\dot{\theta}^2 + m_{sensor} g\cos\theta$ (3)

$F_\theta = -m_{sensor} r\ddot{\theta} - m_{sensor} g\sin\theta$ (4)

The acceleration components, as measured by

$a_\theta = -2\kappa r\omega\theta 0 e^{-\kappa t}\sin(\omega t)$

$+\frac{g \cdot r}{L} - \alpha \; \theta e^{-\kappa t} \cos(\omega t)$ (14)

Some comments can be made about Eqs. (11) and (12). We first focus on the radial acceleration a_r.
the accelerometer at a distance r from the pivot, are then given by:
cos(cos(x)) and sin (x) are both π-periodic
functions. So a_r is a $\frac{T}{2}$ periodic function with

$T = \frac{2\pi}{\omega}$

$a_r = r\dot{\theta}^2 + g\cos\theta$ (5)

$a_\theta = -r\ddot{\theta} - g\sin\theta$ (6)

Including the expression of $\ddot{\theta}$ (Eq. (2)) into the expression of a_θ gives :
ω being the pendulum period. Indeed,
the pendulum reaches its maximum velocity and so its maximum radial acceleration each time θ is equal to 0.

- g cos ($\theta_0 e^{-\kappa t}\cos(\omega t)$) varies between between g cos(θ_0) and g (blue curve in Fig. 4(b)) and goes to g as t approaches infinity. Physically,

$a = \frac{g \cdot r}{L} - \alpha \; \sin\theta + 2\kappa r\dot{\theta}$ (7)

this function represents the projection of $\vec{g}$

onto the pendulum axis *Ott*.

We choose $\theta = \theta_0$ and $\dot{\theta} = 0$ as initial conditions • $r\omega^2\theta_0^2 e^{-2\kappa t}\sin^2(\omega t)$ varies between $r\omega^2\theta_0^2$ and

of the pendulum movement. We only consider here 0 and the upper envelope of this function (red

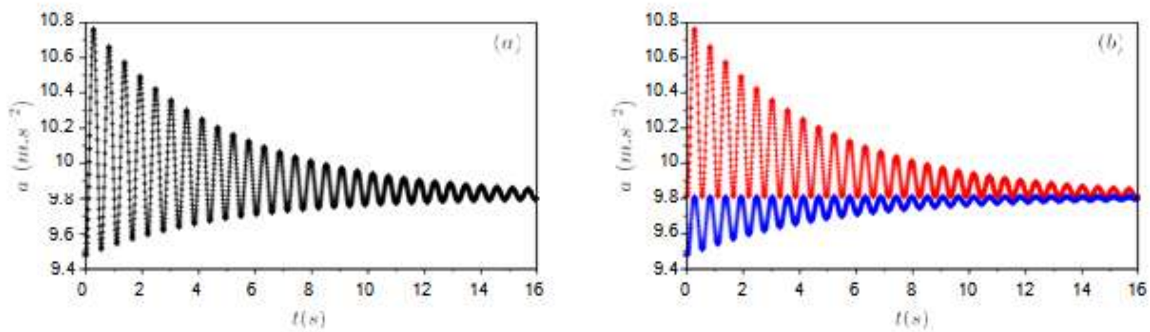

Figure 4: (a) Calculated radial acceleration versus time. (b) In red, graph of the $r\omega^2\theta_0^2 e^{-2\kappa t}\sin^2(\omega t) + \alpha$ contribution; in blue, graph of the $g \cos\left(\theta_0 e^{-\kappa t}\cos(\omega t)\right)$ contribution. $L = 30$ cm, $\theta_0 = 15°$, $r = 45$ cm, $\kappa = 0.1\ s^{-1}$ and $\alpha = 1$ for both panels.

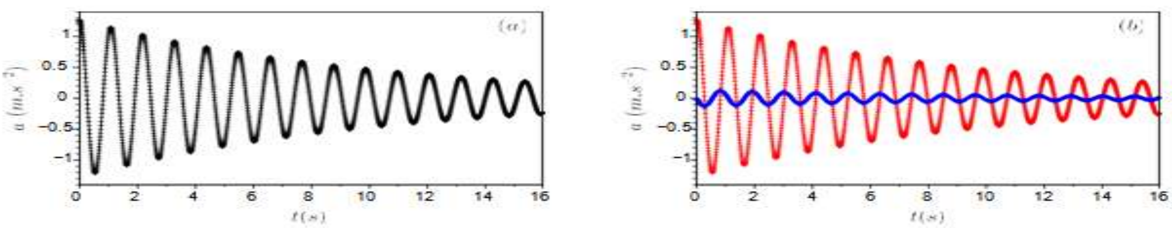

Figure5:(a) Calculated orbital acceleration versus time. (b) In red, graph of the $\frac{g \cdot r}{L} - \alpha \; \sin\theta_0 e^{-\kappa t}\cos(\omega t)$ contribution; in blue, graph of the part $-2\kappa r\omega\theta_0 e^{-\kappa t}\sin(\omega t)$. L=30cm, $\theta_0 = 15°$, r=45cm, $\kappa = 0.1\ s^{-1}$ and $\alpha = 1$ for both panels.

curve in Fig. 4(b), note that it has been dis- placed by g) decreases exponentially as $e^{-2\kappa t}$. Physically,thisfunctionrepresentstheacceler- ation due to the radial centrifugal force feltby thesensor(asexpected,theaccelerationdueto the radial centrifugal force is maximum when the pendulum is vertical). This point and the previousoneexplainstheasymmetryofthera- dialaccelerationaboutthestraightlinea_r=g, as observed in Fig.2.

We can now study the orbital acceleration a_θ:

- sin(cos(x)) and sin(x) are both2π-periodic functions. Therefore, $a_\theta$is a T -periodicfunc-tion with T = $\frac{2\pi}{}$. Indeed, the angular ac- celeration$\ddot{\theta}$reachesitshighestandlowestval- ueseachtimethependulumpassesthroughits highest points. This point explains that the period of the radial acceleration is twice that of the orbitalone.

$\frac{gr}{\alpha}$ sin ($\theta 0 e^{-\kappa t}$cos(ωt)) goes to 0 as t approaches infinity and the sin(cos(x)) func-tionimpliesthattheupperandlowerenvelope of this part of the $a_\theta$function are symmetric about the straight line of equation a = 0 (red curve in Fig.5(b)).

- $-2\kappa r\omega\theta 0 e^{-\kappa t}$sin(ωt) goes to zero as t

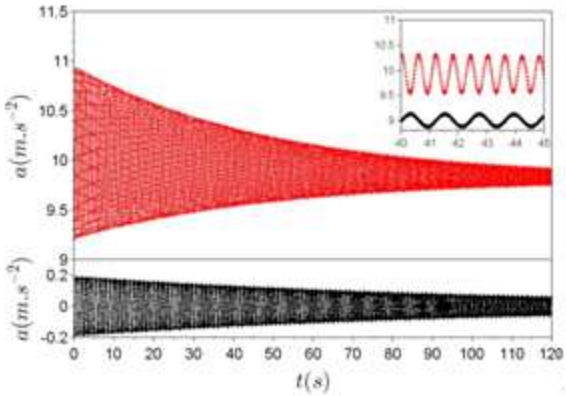

Figure 6: Radial (red) and orbital (black)accelerations

obtained from Eqs. (11) and (12) with $\theta 0$ = 20°, L = 30 cm, r = 34 cm, α = 1.2 and κ = 0.01 s^{-1}. Inset showsthetemporalevolutionofradial(red)andorbital (black) accelerations between 40 s and 45 s. In order to havebothcurvesonthesameplot,orbitalacceleration

is shifted by a constant offset of 9 in the inset.

ThesensorpositionOAcanbemeasuredwith great accuracy. Thus, the position of the pen- dulum center of mass and moment of inertia are the two quantities which are difficult to determineexperimentally. Here,weusethefit of the temporal evolution of $a_r$and $a_\theta$to de- termine the productαL.

iii. Impactofαandronthemeasuredra- dial and orbitalaccelerations

Figure 7 displays the evolution of the radial and orbital accelerations with time for different values of the moment of inertia (from α = 1 (Panel (a)) to α = 2.5 (Panel (d))). At the difference of the radial acceleration, we can see that the orbital accelera- tion depends strongly on α. Indeed, $a_\theta$expression at t = 0 leads to a (t = 0) =$\frac{r}{}$ − 1 g sin θ,

which is an inverse function of α. We can also note

that a_θ(t = 0) <0 if α >$\frac{r}{}$and a_θ(t = 0) >0 if θ αL 0
L
α <$\frac{r}{}$

approaches infinity. The upper and lower en- velopesofthispartofthe$a_\theta$functionaresym- metric about the straight line a_θ= 0 (blue curve in Fig. 5(b)) and decreaseexponentially as$e^{-\kappa t}$.

Figure 6 displays the radial (red) and orbital (black) accelerations obtained from Eqs. (11) and (12). We can see that the main features inferred from the data (Fig. 2) are well reproduced.

ii. Experimentally accessiblequantities

The parameters describing the pendulum and its motion can be derived from the measurements of $a_r$and a_θ.

- $a_r(t = 0) = a_{r,min}= g \cos \theta 0$. Thus, the mea- suredvalueofa$_r$att=0leadstothevalueof θ0.

A fit to the measured $a_r$upper envelope with anexponentialfunctionallowsustodetermine the damping factor κ of the pendulum from Eqs. (11) and (12). While, for small deflection angles θ, see Eqs. (13) and (14), exponential fitofanyenvelopeofmeasureda$_r$ora$_\theta$allows us to determine the dampingfactor.

L . Thus, value of $a_\theta$at t = 0 gives information on the α value.

Figure 8 shows the evolution of the radial and orbital accelerations with time for different values of r. The distance OA increases from Panel (a) to Panel (d). As expected, the amplitude of the radial acceleration increases with larger OAvalues as the centrifugal force acting on the proof mass increases and the amplitude of the orbital acceler- ation decreases with larger OA values as the rate of variation of $\dot{\theta}$ decreases with thisdistance.

In particular, acceleration components measured bytheaccelerometerattachedtothepositionofthe point O are givenby:

$a_r= g \cos\theta 0 e^{-\kappa t}\cos(\omega t)$ (15) . Σ

$a_\theta= -g \sin\dot{\theta} 0 e^{-\kappa t}\cos(\omega t)^{\Sigma}$ (16)

Inthiscase,theonlyforceactingonthemassinside theaccelerometerisitsweightandtheexpressions oftheaccelerationdonotdependonα.Accelerom- eter sensor is used in this case as an angular posi- tionsensor.

Wealsonotethattheaccelerationcomponents measuredbytheaccelerometerattachedtothepo- sition r = Lα are givenby:

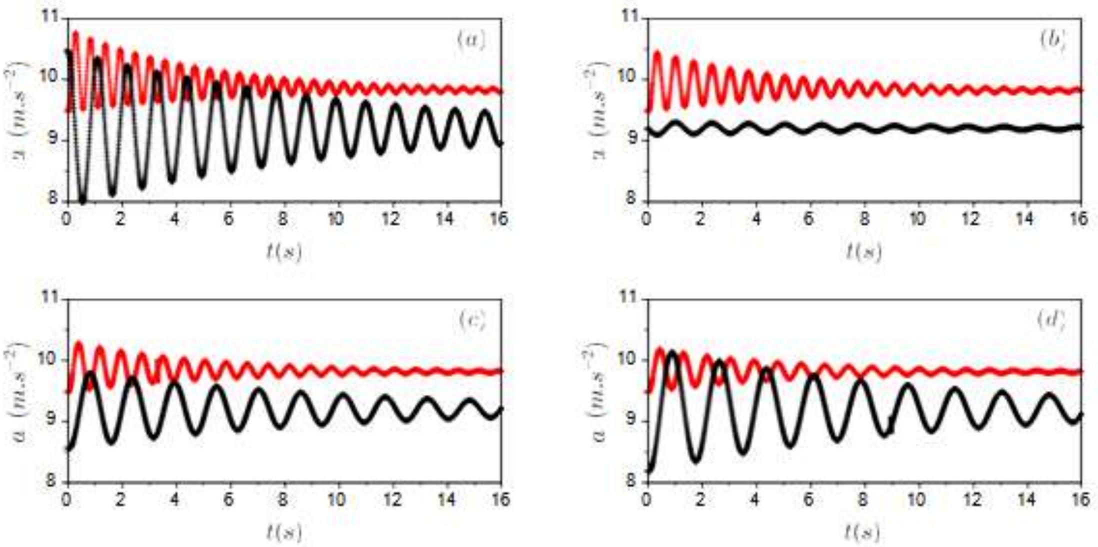

Figure 7: The calculated radial (red) and orbital (black) acceleration with time with α = 1 (a), α = 1.5 (b), α = 2.0 (c) and α = 2.5 (d). In order to have both curves on the same plot, orbital acceleration is shifted by a constant offset of 9.2. L = 30 cm, θ0 = 15°, r = 45 cm and κ = 0.1 for allpanels.

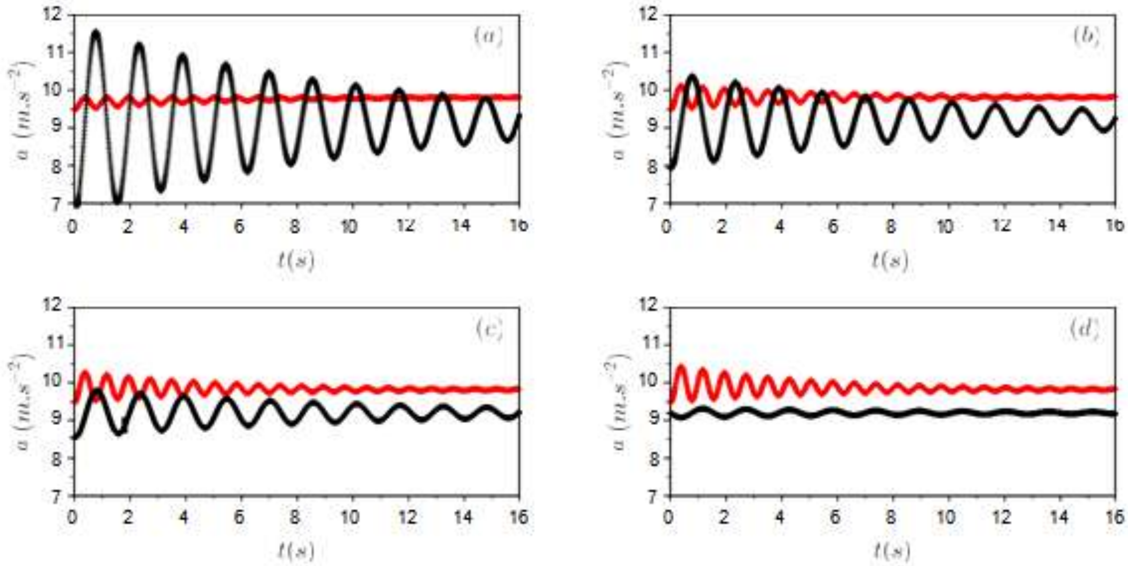

Figure 8: The calculated radial (red) and orbital (black) accelerations with time for different positions of the accelerometer relative to the center of mass tt. In order to have both curves on the same plot, the orbital acceleration is shifted by a constant offset of 9.2. The accelerometer positions are (a) r = 0 cm, (b) r = 30 cm,
(c) r = 45 cm, (d) r = 60 cm. L = 30 cm, $\theta_0 = 15^{\circ}$, $\alpha = 2$ and $\kappa = 0.1$ for allpanels.

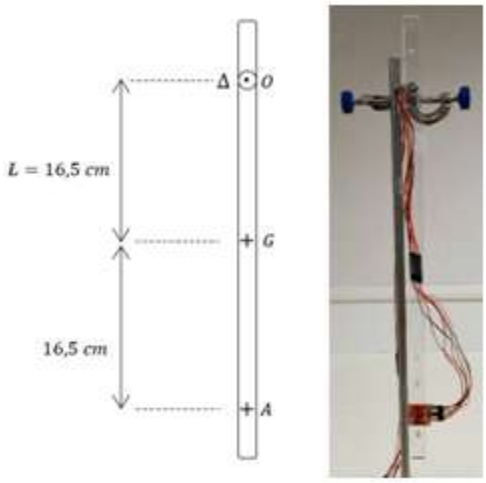

Figure 9: Experimental setup used to study a bar pen- dulum.

In this case, component of $\ddot{\theta}$due to the gravity force is counterbalanced by the orbital component of the force of gravity acting on the accelerometer sensor. Thus, $a_\theta$is then directly proportional to the pendulum angular velocity.

After having shown that Eqs. (11) and (12) ex- plainthefeaturesobservedexperimentally,wewill nowusethemtoretrievethependulumparameters κ, α and L for different experimentalsetups.

IV. EXAMPLE OF DATAANALYSIS

i. Example with a pendulumbar

We first focus on the pendulum shown in Fig. 9 to derive its physical parameters. We use a bar of a 45 cm length and mass 45 g with the pivotat16.5cmfromthecenterofmassandtheacceleromom- eter at 33cm.

Figure 10 shows the radial and orbital accelera- tionsmeasuredbytheaccelerometerafterthepen- dulumhasbeendisplacedfromtheequilibriumpo- sitiontoaninitialangleof22°.

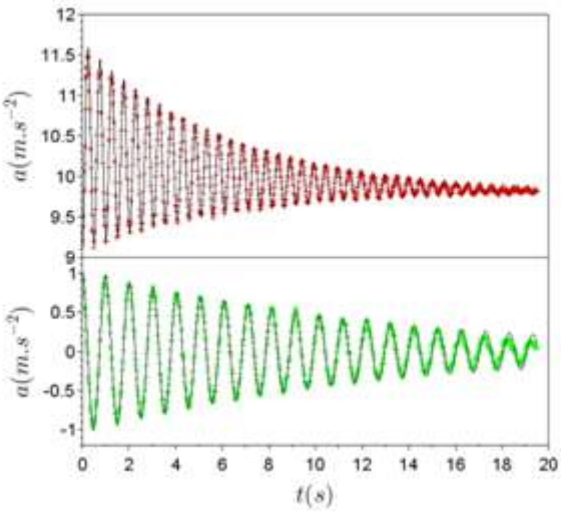

Figure 10: Radial (red) and orbital (green) accelera- tionsmeasuredbytheaccelerometerintheconfigura- tion shown in Fig. 9. Solid lines are the best fit of the data with κ and α as free parameters. The best fit is given by 1/κ = 12.2 s and α =1.56.

We have analyzed the data using Eqs. (11) and (12) with κ and α as free parameters. The results of the fit are shown as black curves in Fig. 10. We have derived the best fit for 1/κ = 12.2 s and α =1.56.
Assumingthependulumtobeasimplehomoge- neous slab, we calculate that the inertia moment of the bar about the rotation axis λ is equalto
1.93 $\times$ 10¯3 kg m2, which leads to α = 1.62.The holes in the bar, which are used for attaching the masses, together with the mass of the accelerom- eter, explain the difference between this and with the α value obtained from the data fit. This exper- imentallowsustodeterminewithagoodaccuracy the moment of inertia of thependulum.

ii. Retrieval of the mass centerposition

Thependulumofthepresvioussubsectionisasym- metric bar, thus, its mass center position can be determinedprecisely.Inthegeneralcase,themass centerpositioncanbedifficulttodetermineandwe can fit the data by using Eqs. (11) and (12) with κ, α and L as free parameters and infer the position of the center ofmass.
As an example of such analysis, we use a

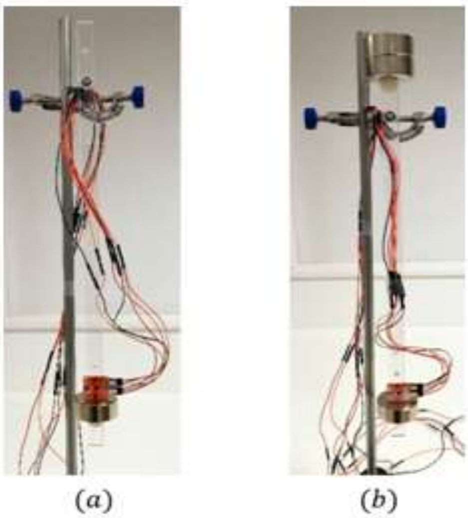

Figure 11: Experimental setup used to study compound pendula, with added masses.

pendulumcomposedofabaronwhichseveralmasses canbeattachedtoatdifferentpositionstoacquired data that corresponds to pendulums of different valuesofαandL(Fig.11(a)and(b)).

Figure 12 (a) and (b) displays the radial and or- bital accelerations measured by the accelerometer after each pendulum shown in Fig. 11 has been displaced by an initial angle of 20.5°.
We fit the data by using Eqs. (11) and (12) with κ, α and L as free parameters. Results of the fitare shown as the black curves in the insets of Fig. 12. Best fit are obtained with 1/κ = 111 s, α = 1.162 and L = 0.293 m for the pendulum of Fig. 11(a) and 1/κ = 101 s, α = 6.55 and L = 0.074 m for the pendulum of Fig.11(b).
For the examples displayed in Fig. 12, the evo- lutionoftheorbitalaccelerationfromPanels(a) to (b)showsanincreaseofα,whichisconsistentwith the fact that pendulum configuration goes from a configuration close to a simple pendulum (α 1Fig. 11(a)) to a compound pendulum (α >1 Fig. 11(b)).
Figure12showsthattheangularacceleration(in green)ismuchmoresensitivetothevalueofαthan the radial acceleration (in red). Therefore, orbital accelerationisagoodquantitytomeasureandtofit in order to determine the moment of inertia of a pendulum.

iii. Impact of r on the acquireddata

WehaveshowninsectionIIIthattheorbitalaccel- eration is very sensitive to the accelerometer sen- sorpositionwithrespecttothependulumrotation axis. As this position is precisely known, we can performseveralmeasurementswithdifferentposi- tionsoftheaccelerometertoimprovetheaccuracy and/or the precision of the derived pendulum pa- rameters. As an example of such analysis, we use the experimental setups shown in Fig.13.

Figure 14 (a) and (b) display the radial and or- bital accelerations measured by the accelerometer after each pendulum shown in Fig. 13 has been displaced by an initial angle of 20.5°.
Fits of the data using Eqs. (11) and (12) with κ, α and L as free parameters are shown as black curves in Fig. 14. Best fit are obtained with 1/κ = 111 s, α = 1.162 and L = 29.3 cm for the pendulum of Fig. 13(a) and with 1/κ = 100 s, α = 1.162 and L = 29.2 cm for the pendulum of Fig.13(b).

The position of the accelerometer does not af- fect the calculated values of the moment of inertia of the pendulum and only slightly affects the cen- ter of mass position. The new configuration of the wires connecting the accelerometer in Fig. 13(b) changes slightly the κ value. Thus, performing a secondmeasurementwithadifferentpositionofthe accelerometerallowustobemoreconfidentinthe results retrieved from the firstone.

We can also note that the orbital acceleration increases with lower OA values, therefore the or- bital acceleration fit precision is better when the accelerometerisinthepositionofFig.13(b),while the precision of the radial acceleration fit is better when the accelerometer is in the position of Fig. 13(a).

V. CONCLUSIONS

Wehaveshownthatthependulumexperimentana- lyzedwithanaccelerometersensorleadstoatheo- retical study richer than the classical one. We have derived theoretical expressions for the radial and orbitalaccelerationdatarecordedbyanaccelerom- eterandseparatedthecontributionsfromthepen-

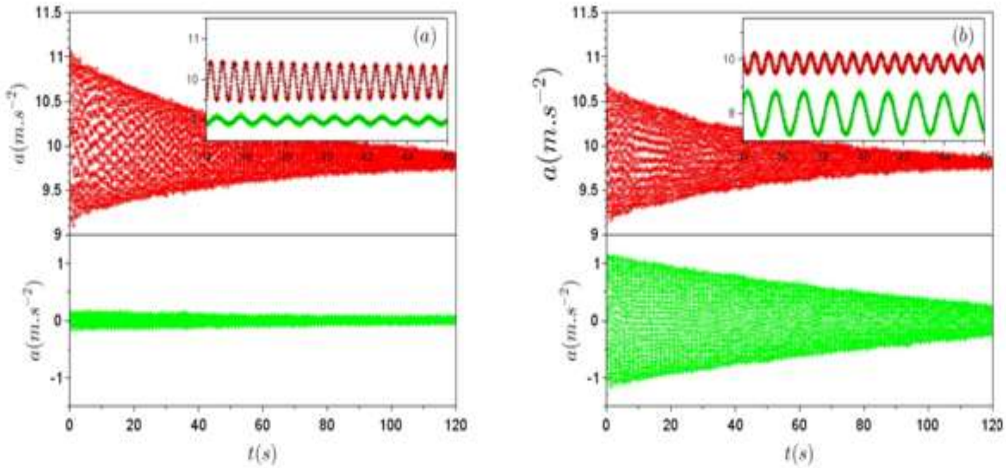

Figure 12: Radial (red) and orbital (green) accelerations measured by the accelerometer. (a) and (b) correspond to the setups displayed in Fig. 11 (a) and (b), respectively. Insets in (a) and (b) show the temporal evolution of the radial (red) and orbital (green) accelerations and the best fit (black) between 35 s and 45 s. In order to have both curves on the same plot, orbital acceleration is shifted by a constant offset of 9 in the inset of (a) and by 8 in(b).

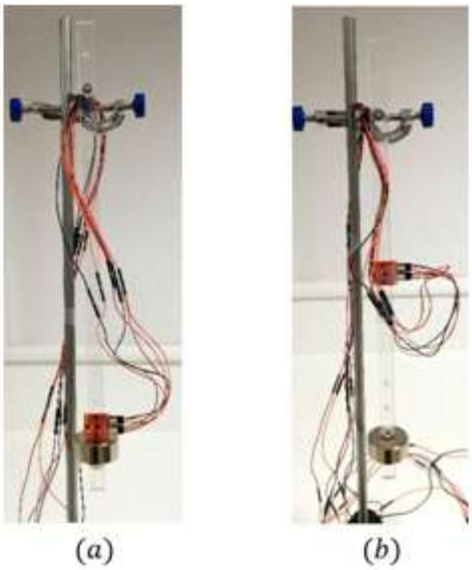

Figure 13: Experimental setup used to study the im- pact of the acceleration sensor position on the acquired data.

dulum angular motion and the gravitational force on the proof mass.

We have also shown that the orbitalacceleration is an interesting data to retrieve the moment of inertia of a pendulum.

The possibility to have different positions of the sensorallowsustoperformseveralmeasurements with the same pendulum to improve the accuracy and/or the precision of the derived pendulum parameters.

In this paper, we have only focused on the clas- sicalpendulumbutthedeviceusedherecouldalso beappliedtomorecomplexsystemssuchaschaotic pendulums.

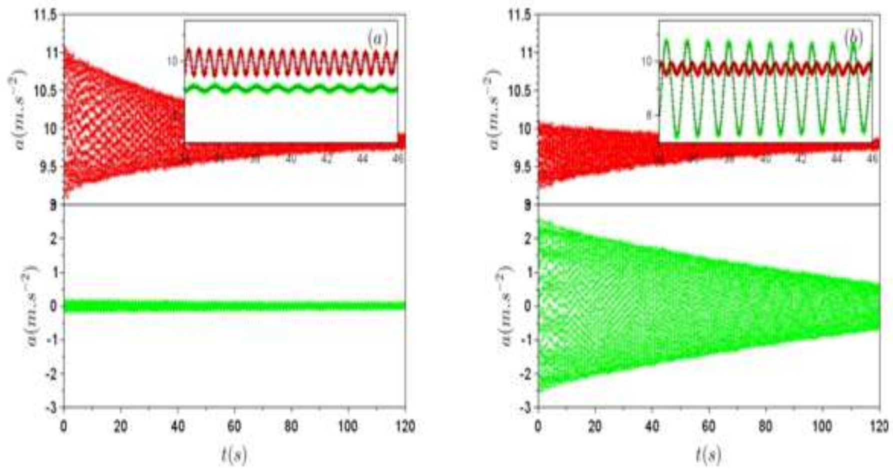

Figure 14: Radial (red) and orbital (green) accelerations measured by the accelerometer. (a) and (b) correspond to the setups displayed in Fig. 13(a) and (b), respectively. Insets in (a) and (b) show the temporal evolution of the radial (red), and orbital (green) accelerations and the best fit (black) between 35 s and 45 s. In order to have bothcurvesonthesameplot,orbitalaccelerationisshiftedbyaconstantoffsetof9intheinsetsof(a)and(b).

REFERENCE

[1] http://www2.vernier.com/sample_labs/PHYS-AM-18-physical_pendulum.pdf
[2] https://physics.fullerton.edu/files/ Labs/225/Physical_Pendulum_V4_1(1).pdf
[3] JCFernandes,PJSebastião,LNGonçalves,A Ferraz, Study of large-angle anharmonic os- cillations of a

physical pendulum using an ac- celeration sensor, Eur. J. Phys. **38**,2017.
[4] https://www.arduino.cc
[5] ŠKubínová, JŠlégr,Physicsdemonstrations with the Arduino board, Phys. Educ. **50**, 472 (2015).
[6] C Galeriu, S Edwards, G Esper, An Ar- duino investigation of simple harmonic mo- tion, Phys. Teach. **52**, 157(2014).
[7] K Atkin, Construction of a simple low-costtes- lameter and its use with Arduino and Maker- Plot software, Phys. Educ. **51**, 024001(2016).
[8] C Petry et al., Project teaching beyond Physics: Integrating Arduino to the laboratory, InTechnologiesAppliedtoElectronicsTeach- ing (TAEE), pp. 1-6, IEEE(2016).
[9] RHenaffetal.,Astudyofkineticfriction:The Timoshenko oscillator, Am. J. Phys. **86**, 174 (2018).
[10] F Bouquet, J Bobroff, M Fuchs-Gallezot, L Maurines, Project-based physics labs using low-cost open-source hardware, Am. J. Phys. **85**, 216(2017).